# Dib's Fuzzy Hv-ideals in fuzzy $H_V$ -semigroups based on fuzzy space

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**الملخص:**

الفكرة المعتمدة لهذا البحث هي دراسة وتوضيح بعض المفاهيم والخواص الأساسية للمثاليات الضبابية العليا، الخاصة بشبه الزمر الضبابية التي تعتمد على الفضاء الضبابي المُعرّف بواسطة العالم الرياضي "ديب". كما سنوضح العلاقة بين هذه المثاليات العليا الضبابية (اليمنى، اليسرى) والمثاليات الضبابية الكلاسيكية المعتمدة على المجموعات الضبابية.

## Abstract:

The aim of this paper is to define and study the concept of a fuzzy (left, right) hyperideal in fuzzy semihypergroup, which depends on the concept of a fuzzy space as a direct generalization of the concept of fuzzy ideals in fuzzy semigroups, through the new approach of fuzzy space introduced by (Dib. K, 1996) and fuzzy hypergroup based on fuzzy space. A correspondence relationship between the introduced fuzzy (left, right) hyperideals and fuzzy (left, right) hyperideals based on fuzzy universal set by (Davvaz. B, 1998) is established.

**Keywords** Hyperideals, Fuzzy space, Fuzzy universal sets, Fuzzy hyperoperation, Fuzzy hypergroups, Fuzzy semihypergroup, Fuzzy ideals, Fuzzy hyperideals, $H_V$ – ideals.

## 1 Introduction and basicconcepts

The fuzzy Logic uses the whole interval between 0 (false) and 1 (true) to describe human reasoning. A Fuzzy Set is any set that allows its members to have different degree of membership, called membership function, having interval [0, 1]. In (Marty. F, 1934) introduced the notation of Hyperstructure theory by his definition of the hypergroups, the theory of hyperstructure introduced by Marty is a new mathematical structure which represents a natural extension to the classical algebraic structure. In classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Formally, if H is a nonempty set and $P^*(H)$ is the set of all nonempty subsets of $H$ then we consider maps of the following type: $f_i : H \times H \rightarrow P^*(H)$ where $i = 1, 2, \ldots, n$ and $n$ is a positive integer. The maps $f_i$ are called (binary) hyperoperations. A hypergroup $H$ is a hyperstructure $(H, \circ)$ satisfying;

1. $x \circ (y \circ z) = (x \circ y) \circ z$ for all $x, y, z \in H$, (1)

2. $x \circ H = H \circ x = H$ for all $x \in H$. (2)

If $(H, \circ)$ satisfies only condition (1) then $(H, \circ)$ is called a semihypergroup and if we replace condition (1) by $x \circ (y \circ z) \cap (x \circ y) \circ z = \varphi$ (which is obviously weaker than condition (1)), then the hyperstructure $(H, \circ)$ is called an $H_V$-group. Zadeh in (Zadeh. L, 1965) introduced notation of the fuzzy set theory generalizes classical set theory in a way that membership degree of an object to a set is not restricted to the integers 0 and 1, but may take on any value in [0, 1]. Therefore a fuzzy subset A of a set $X$ is a function $A : X \to [0, 1]$, usually this function is referred to as the membership function and denoted by $\mu_A(x)$. Some mathematicians use the notation $A(x)$ to denote the membership function instead of $\mu_A(x)$. A fuzzy subset $A$ is written symbolically in the form $A = \{(x, \mu_A(x)) : x \in X\}$. By starting with a given group, Rosenfeld (Rosenfeld. A, 1971) defined a fuzzy subgroup of the given group using the binary operation defined over the group. More precisely, if $G$ is a group with binary operation $\circ$ and $A$ is a fuzzy subset of $G$, then $A$ is called a fuzzy subgroup of $G$ (in the sense of Rosenfeld) *if* for all $x, y \in G$,

1. $A(x \circ y) \geq \min\{A(x), A(y)\}$, (3)
2. $A(x^{-1}) \geq A(x)$. (4)

Thus, the binary operation on the fuzzy subgroup $A$ is defined in terms of the ordinary binary operation defined over the ordinary group $G$ and the study of the fuzzy algebraic structures was introduced and studied with the concept of fuzzy subgroups by Rosenfeld. In many cases, the connection between such algebras and their corresponding logics is much stronger. In (Liu. J 1982), (Liu. J, Guilong. L, 2007), Liu introduced the concept of a fuzzy ideal of a ring. The study of fuzzy structures is an interesting research topic of fuzzy sets, hundreds of papers and several books have been written on fuzzy structure, for example see (Corsini. P, 2002), (Davvaz. P, 2006), (Guilong.L, 2007), (Guilong.L, 2006), (Kazanci. O, 2008), (Kazanci. O, 2009), (Marius. T, 2009), (Marius. T, 2009), (Rajesh. K, 1992), (Xueling. M. et al, 1992), and the study continued to fuzzy hyperstructure by (Davvaz. B. et al, 2019, 1998, 1999, 2010(a), 2010(b), 2020 ), he with Leoreanu-Fotea (Leoreanu-Fotea. V, Davvaz. B, 2008), and (Leoreanu-Fotea. V, 2009), also see (Sen. M. et al, 2008). The main problem in fuzzy mathematics is how to carry out the ordinary concepts to the fuzzy case. The difficulty lies in how to pick out the rational generalization from the large number of available approaches. Davvaz (Davvaz. B, 1999) introduced the notions of fuzzy hypergroup as generalization of Fuzzy subgroupoids and fuzzy subgroup by (Rosenfeld. A, 1971) based on the notion of fuzzy set by (Zadeh. L, 1965). That is he assumed a given ordinary hypergroup and define a fuzzy subhypergroup of the given ordinary hypergroup. Again such formulation of the intrinsic definition for fuzzy hypergroup and fuzzy subhypergroup is not evident due to the absence of the concept of fuzzy universal set, where he applied in fuzzy sets to the theory of algebraic hyperstructures and defined fuzzy subhypergroup (respectively $H_V$-subgroup) of a hypergroup (respectively $H_V$ -group) which is a generalization of the concept of Rosenfeld’s fuzzy subgroup of a group. Let $(H, \circ)$ be a hypergroup (respectively, $H_V$ -group) and let $\mu$ be a fuzzy subset of $H$. Then $\mu$ is said to be a fuzzy subhypergroup (respectively, fuzzy $H_V$ -subgroup) of $H$ if the following axioms hold:

1. $min\{\mu(x), \mu(y)\} \leq inf\{\mu(z) : z \in x \circ y\}$ for all $x, y \in H$, (5)
2. For all $x, a \in H$ there exists $y \in H$ such that $x \in a \circ y$ and
   $min\{\mu(a), \mu(x)\} \leq \mu(y)$, (6)
3. For all $x, a \in H$ there exists $z \in H$ such that $x \in z \circ a$ and
   $min\{\mu(a), \mu(x)\} \leq \mu(z)$. (7)

Similar to the case of fuzzy algebra, fuzzy hyperalgebra follows the same approaches in generalizing the notions of hypergroup and hyperring. Some mathematicians criticized the notion of Zadeh’s fuzzy set, Davvaz defined a new kind of fuzzy ideal and fuzzy hyperideal which is based on the notation of fuzzy hypergroup, fuzzy hyperring and fuzzy hyperideal based on Rosenfeld"s approach (Rosenfeld. A, 1971). The study of fuzzy hyperalgebraic structures has started with the introduction of the concepts of fuzzy (hypergroups) subhypergroups, fuzzy semihypergroups, fuzzy hyperring, fuzzy hyperideal based on fuzzy universal set by (Davvaz. B. et al, 1998, 1999). The relation between fuzzy hypergroup, fuzzy hyperring based on fuzzy space of (Dib. K, 1994) with fuzzy hypergroup and fuzzy hyperring based on fuzzy universal sets was introduced by (Davvaz. B. et al, 2010(a), 2010(b)). Where Dib introduced the notion of fuzzy space which play the important role of universal set in ordinary classical mathematics. Through thea new approach of fuzzy spaces and fuzzy groups introduced by (Dib. K, 1994), He and Galhum in (Dib. K. Galhum. N, 1996) introduced some basic concepts of fuzzy algebra, such as fuzzy (left, right) ideals in fuzzy semi group and studied the relation between the introduced fuzzy (left, right) ideals and classical ones. Dib (Dib. K, 1994) defined a fuzzy space $(X, \mathbf{I})$ is the set of all ordered pairs $(x, \mathbf{I})$, $x \in X$; i.e., $(X, \mathbf{I}) = \{(x, \mathbf{I}) : x \in X\}$, where $(x, \mathbf{I}) = \{(x, r) : r \in \mathbf{I}\}$. The ordered pair $(x, \mathbf{I})$ is called a fuzzy element in the fuzzy space $(X, \mathbf{I})$. Let $U_\circ$ denote the support of $U$, that is $U_\circ = \{x : A(x) > 0\}$. A fuzzy subspace $U$ of the fuzzy space $(X, \mathbf{I})$ is the collection of all ordered pairs $(x, u_x)$, where $x \in U_\circ$ for some $U_\circ \subset X$ and $u_x$ is a subset of $\mathbf{I}$, which contains at least one element beside the zero element. If it happens that $x \notin U_\circ$, then $u_x = 0$. An empty fuzzy subspace is defined as $\{(x, \varphi_x) : x \in \varphi\}$. Let $U = \{(x, u_x) : x \in U_\circ\}$ and $V = \{(x, v_x) : x \in V_\circ\}$ be fuzzy subspaces of $(X, \mathbf{I})$. The union and intersection of $U$ and $V$ are defined respectively as follows:

$$U \cup V = \{(x, u_x \cup v_x) : x \in U_\circ \cup V_\circ\} \tag{8}$$
$$U \cap V = \{(x, u_x \cap v_x) : x \in U_\circ \cap V_\circ\} \tag{9}$$

The use of fuzzy space as a universal set corrects the deviation in the definition of fuzzy group (hypergroup) theory. Dib (1994) introduced fuzzy group based on the definition of fuzzy space by defining fuzzy structure (fuzzy groupoid). More precisely, a fuzzy space $(X, \mathbf{I})$ together with a fuzzy binary operation $\underline{F} = (F, f_x)$ is said to be a fuzzy groupoid and is denoted by $((X, \mathbf{I}); \underline{F})$. As we know, the concept of fuzzy space played an important role in fuzzy mathematics, the main problem in fuzzy mathematics is how to carry out the ordinary fuzzy concepts to the fuzzy case. The concept of a fuzzy space was introduced in (Dib. K, 1994), as a generalization of the notion of fuzzy sets, and in (Fathi. M, 2010) introduced a new formulation of intuitionistic fuzzy groups by introducing the concepts of intuitionistic fuzzy space and intuitionistic fuzzy functions. Davvaz used Rosenfeld's idea to establish the fuzzification of the concept of hyperideals in a semihypergroup. (Abdulmula. K. et al, 2013), (Abdulmula. K, 2021) introduced the concept of intuitionistic fuzzy hyperalgebra based on intuitionistic fuzzy space. In this paper, we introduce some basic concepts of fuzzy hyperalgebra, namely fuzzy (left, right) hyperideals in fuzzy semihypergroups, though the new approach of fuzzy spaces and fuzzy hypergroups as direct generalisation of the fuzzy (left, right) ideals in fuzzy semigroups by (Dib.K, Galhum. N, 1996). A relation between the introduced fuzzy (left, right) hyperideals based on fuzzy space and fuzzy (left, right) hyperideals based on fuzzy universal sets (the classical case) by Davvaz are given. Moreover, we give some examples to illustrate the difference between our point of view and the classical case.

## 2 Preliminaries

Here we recall some of the fundamental concepts and definitions required in the sequel.

**Definition 1** (Dib. K, 1949) A fuzzy space $(X, I)$ is the set of all ordered pairs $(x, I)$; $x \in X$, $(X, I) = \{(x, I) : x \in X\}$, where $(x, I) = \{(x, r) : r \in I\}$.

**Definition 2** (Dib. K, 1949) A fuzzy subspace U of the fuzzy space $(X, I)$ is the collection of all ordered pairs $(x, u_x)$, where $x \in U_\circ$ for some $U_\circ \in X$ and $u_x$ is a subset of $I$, which contains at least one element beside the zero element. If it happens that $x \notin U_\circ$, then $u_x = 0$. An empty fuzzy subspace is defined as $\{(x, \emptyset x): x \in U_\circ\}$. For a fuzzy subset $A$ of X, A induces the following fuzzy subspace $H(A)$ (called the induced fuzzy subspace by $A$) of $X$ : $H(A) = \{(x, [0, A(x)]), x \in A_\circ\}$, where $A_\circ = \{x \in X, A(x) = 0\}$ is the support of $A$.

**Definition 3** (Dib. K, 1949) A fuzzy binary operation $F = (F, f_{xy})$ on the fuzzy space X is a fuzzy function F from $X \times X$ to X with comembership functions $f_{xy} : I \times I \longrightarrow I$ that satisfy,

1. $f_{xy}(r, s) = 0$ iff $r = 0$ or $s = 0$, (10)

2. $f_{xy}$ are onto , i.e., $f_{xy}(I, I) = I$ for all $(x, y) \in X \times X$. (11)

Recall that the action of the fuzzy function $\blacklozenge = (M, O_{xy})$ on fuzzy elements of the fuzzy space $(H, I)$ can be symbolized as follows:

$$(x, I)\blacklozenge(y, I) = (x \, M \, y, O_{xy}(I, I)) = (M(x, y), I). \quad (12)$$

**Definition 4** (Davvaz. B. et al, 2010) Let $((X, I), F)$ be a fuzzy group having the fuzzy subgroup $U = \{(x, u_x); x \in U_\circ\}$. Contrary to the ordinary case, the fuzzy elements

$(x, u_x)$ of the fuzzy subgroup U are not necessary associative with fuzzy elements $(x, I)$ of the fuzzy group $((X, I), F)$ in the usual sense. That is, $\alpha F(\beta F\gamma) = (\alpha F\beta)F\gamma$, where $\alpha, \beta$ and $\gamma$ are some fuzzy elements of U or $(X, I)$ such that one or two of $\alpha, \beta$ or $\gamma$ belong to U .

**Definition 5** (Davvaz. B, 1999) A hypergroupoid $(H, *)$ is called a semihypergroup if

$(x * y) * z = x *(y * z)$ for all $x, y, z \in H$ .

Davvaz (Davvaz. B, 1998, 1999) introduced the notions of fuzzy hypergroup, fuzzy $H_v$-hyperring and fuzzy $H_v$-ideal as generalization of Fuzzy subgroupoids and fuzzy subgroup by Rosenfeld in 1971 based on the notion of fuzzy set by Zadeh in 1965. That is he assumed a given ordinary hypergroup (hyperring) and define a fuzzy subhypergroup (subhyperring) of the given ordinary hypergroup (hyperring). Again such formulation of the intrinsic definition for fuzzy hypergroup (hyperring) and fuzzy subhypergroup (subhyperring) is not evident due to the absence of the concept of fuzzy universal set, where he applied in fuzzy sets to the theory of algebraic hyperstructures and defined fuzzy subhypergroup (respectively $H_v$-subgroup) of a hypergroup (respectively $H_v$ -group) which is a generalization of the concept of Rosenfeld's fuzzy subgroup of a group. Let $(H, \circ)$ be a hypergroup (respectively,

$H_V$-group) and let $\mu$ be a fuzzy subset of $H$. Then $\mu$ is said to be a fuzzy subhypergroup (respectively, fuzzy $H_V$-subgroup) of $H$ if the following axioms hold:

1- $min\{\mu(x), \mu(y)\} \leq \inf\{\mu(z) : z \in x \circ y\}$ for all $x, y \in H$, (13)
2- for all $x, a \in H$ there exists $y \in H$ such that $x \in a \circ y$ and
$min\{\mu(a), \mu(x)\} \leq \mu(y)$, (14)
3- for all $x, a \in H$ there exists $z \in H$ such that $x \in z \circ a$ and
$min\{\mu(a), \mu(x)\} \leq \mu(z)$. (15)

**Definition 6** Let $R$ be a hyperring and let $A$ be a fuzzy subset of $R$. Then, $A$ is said to be a left (respectively, right) fuzzy hyperideal of $R$ if the following axioms hold:
1. $min\{A(x), A(y)\} \leq inf\{A(\alpha)\}$, for all $x, y \in R$, (16)
2. for all $x, a \in R$ there exists $y \in R$ such that $x \in a + y$ and $min\{A(x), A(a)\} \leq A(y)$; (17)
3. for all $x, a \in R$ there exists $z \in R$ such that $x \in a.z$ and $min\{A(x), A(a)\} \leq A(z)$, (18)
4. $A(y) \leq \inf\{A(\alpha)\}$ (respectively $A(x) \leq \inf\{A(\alpha)\}$ for all $x, y \in R$. (19)

**Definition 7** (Davvaz. B. et al, 2010) Let (H,I) be a non-empty fuzzy space. A fuzzy hyperstructure (hypergroupoid), denoted by $((H, I), \blacklozenge)$ is a fuzzy space together with a fuzzy function having onto co-membership functions (referred as a fuzzy hyperoperation) $\blacklozenge : (H, I) \times (H, I) \rightarrow P^*(H, I)$, where $P^*(H, I)$ denotes the set of all nonempty fuzzy subspaces of $(H, I)$ and $\blacklozenge = (M, O_{xy})$ with $M: H \times H \rightarrow H$ and $O_{xy}: I \times I \rightarrow I$. A fuzzy hyperoperation $\blacklozenge = (M, O_{xy})$ on $(H, I)$ is said to be uniform if the associated comembership functions $O_{xy}$ are identical, i.e.,$O_{xy} = O$ for all $x, y \in H$. A uniform fuzzy hyperstructure $((H, I), \blacklozenge)$ is a fuzzy hyperstructure $((H, I); \blacklozenge)$ with uniform fuzzy hyperop- eration. Recall that the action of the fuzzy function $\blacklozenge = (M, O_{xy})$ on fuzzy elements of the fuzzy space $(H, I)$ can be symbolized as follows:
$(x, I) \blacklozenge (y, I)\ (xM\, y, O_{xy}\ (I \times I)) = (M\,(x, y), I)$. (20)

**Theorem 1** (Davvaz. B. et al, 2010) To each fuzzy hyperstructure $((H, I), \blacklozenge)$ there is an associated ordinary hyperstructure $(H, M)$ which is isomorphic to the fuzzy hyperstructure $((H, I), \blacklozenge)$ by the correspondence $(x, I) \leftrightarrow x$. in (Dib. K. et al. 1996, 1998) introduced some basic concepts of fuzzy algebra, such as fuzzy (left, right) ideals in fuzzy semi group and studied the relation between the introduced fuzzy (left, right) ideals and classical ones.

**Definition 8** (Dib. K, Hassan. N, 1998) A fuzzy subspace Uof the fuzzy semigroup $((H, I), \underline{F})$ is said to be fuzzy left (right) ideal if for each $(x, I) \in U$, we have

$((x, I)\ \underline{F}\ (y, I_y) = (xF\, y, \underline{f}_{xy}\ (I, I_y)) \in U$ (21)
$((y, I_y)\ \underline{F}\ (x, I) = (yF\, x, \underline{f}_{yx}(I_y, I)) \in U$. (22)

**Definition 9** ( Davvaz. B, 2006) A hypergroupoid $(H, *)$ is called a semihypergroup if
$(x * y) * z = x * (y * z)$ for all $x, y, z \in H$. (23)

## 3 Fuzzy hyperideals in fuzzy semihypergroup

In this section we introduce some basic concepts of fuzzy hyperalgebra such as fuzzy (left, right) hyperideal in fuzzy semihypergroup, through the new approach of fuzzy hypergroup based on fuzzy space and obtain a relationship between fuzzy

(left, right) hyperideal and ordinary (left, right) hyperideal.

**Remark 1** A fuzzy subspace U is called a fuzzy hyperideal of the fuzzy semihypergroup ((H, I), F), if U is both fuzzy left and right hyperideal.

In the following example we show that fuzzy hyperideal is a fuzzy subhypergroup of the fuzzy semihypergroup.

**Example 3.2** Let H ={*a, b, c, d*}, and let *F* be a fuzzy hyperoperation defined in the following table(1).

| F | a | b | c | d |
|---|---|---|---|---|
| a | {*a*} | {b} | {c} | {d} |
| b | {a} | {a} | {d} | {c} |
| c | {a} | {d} | {a} | {b} |
| d | {a} | {c} | {b} | {a} |

We have *F* is associative on (H, I), then ((H, I), F) is a fuzzy semihypergroup, where F= (F, $f_{xy}$), $f_{xy}$ (r, s) = r ∧ s. Now if U = {(a, [0, 1])}, it is easy to see that the fuzzy subhypergroup (U, F) is a fuzzy hyperideal of the fuzzy semihypergroup ((H, I), F).

Next we introduce a relationship between the fuzzy (subhypergroupoid) subsemihypergroup and the ordinary ones.

**Theorem 3.1** Let U be a fuzzy subspace of the fuzzy (hypergroupoid) semihypergroup ((H,I),F). Then (U,F) is a fuzzy (subhypergroupoid) subsemihypergroup of ((H, I), F) i*ff*

**(i)** ($U_\circ$, F) is a (subhypergroupoid) subsemihypergroup of the ordinary

(hypergroupoid) semihypergroup (H, F ),

**(ii)** for all x, y ∈ $U_\circ$ we have $f_{xy}$ ($I \times I_y$ ) = $I_x F_y$ and ($f_{yx}$ ($I_y \times I$ ) = $I_y F_x$ )

By the above theorem, the following example will show the fuzzy (subhypergroupoid) subsemihypergroup of a fuzzy semihypergroup.

**Example 3.2** Let $H$ = { α, β, γ } and let $F$ be a binary hyperoperation on $H$ defined by the following table (2)

| F | α | β | γ |
|---|---|---|---|
| α | {α | {β | {γ |
| β | {β} | {α | {β |
| γ | {γ} | {β} | {α |

Consider the following onto comembership function $f_{xy} : I \times I \longrightarrow I$;

$$f_{\gamma\gamma}(r,s) = \begin{cases} \dfrac{rs}{1-r\Lambda s}, & rs \prec \dfrac{1}{2} \\ 1 \quad , & rs \geq \dfrac{1}{2} \end{cases}$$

$f_{\alpha\alpha}(r,s) = \dfrac{rs}{2-rs}$, $\quad f_{\alpha\gamma}(r,s) = f_{\gamma\alpha}(r,s) = r\Lambda s$

Then, it is easy to show that ((H, I), F), where F= (F, $f_{xy}$) is a fuzzy semihypergroup. Now, let U = { ( α, [0, 1]), (γ, [0, 0.5] ) }, be a fuzzy subspace of the fuzzy space (H, I). Then

**i.** it is easy to show that the subset $U_O$ = { α, γ } satisfies the axioms of the ordinary subhypergroup of the ordinary semihypergroup (H, F ).

**ii.** In the following, we justify condition (ii) of Theorem 3.1,

$$f_{\alpha\alpha}(I_\alpha, I_\alpha) = [0,1] = I_{\alpha F\alpha} = I_\alpha$$

Then, from (i) and (ii), we see that (U, F ) is a fuzzy subhypergroup of the fuzzy semihypergroup ((H, I), F).

**Theorem 3.3** Let (U, F) be a fuzzy subhypergroup of the fuzzy semihypergroup ((H, I), F ). Then (U, F) is a fuzzy left (right) hyperideal *iff*

**(i)** ($U_O$, F ) is a left (right) hyperideal of the ordinary semihypergroup (H, F ),

**(ii)** for all $x \in$ H, $y \in U_O$ ; $f_{xy}(I \times I_y) = I_{xFy}$ and $(f_{yx}(I_y \times I) = I_{yFx})$.

We would like to point out that the proof of all the theorems in this section are carried out for the fuzzy left hyperideal, and the case of fuzzy right hyperideals is similar.

**Proof 1** Firstly, suppose that the fuzzy subhypergroup (U, F ) is a fuzzy left hyperideal of the fuzzy semihypergroup ((H, I), F). Then by Theorem 3.1, we have

(i) ($U_O$, F ) is a subhypergroup of the ordinary semihypergroup (H, F ),

(ii) for every $y, z \in U_O$ we have $f_{yz}(I_y, I_z) = I_{yFz}$.

Moreover, for every $(x, I) \in (H, I)$, $(y, I_y) \in U$, we have $(x, I)F(y, I_y) = (xFy, f_{xy}(I \times I_y))$

Therefore $(xFy, f_{xy}(I \times I_y)) = (\{z\}, I_z) \subseteq P^*(U)$. (24)

For all $x \in H$, $y, z \in U_O$, Equation (24) is equivalent to

(a) $xFy = \{z\} \subseteq P^*(U_O)$,

(b) $f_{xy}(I \times I_y) = I_z = I_{xFy}$.

Condition (a) means that ($U_O$, F ) is a left hyperideal of the semihypergroup (H, F ) and (b) is (ii).

The next theorem gives a necessary and sufficient condition for the concept of a fuzzy hyperideal in fuzzy semihypergroup by illustrates the relation between these concepts and the ordinary ones.

**Theorem 3.4** Let $(U, F)$ be a fuzzy subhypergroup of the fuzzy semihypergroup $((H, I), F)$. Then $(U, F)$ is a fuzzy hyperideal of $((H, I), F)$ *iff*

(i) $(U_\circ, F)$ is a hyperideal of the ordinary semihypergroup $(H, F)$,

(ii) for all $x \in H$, $y \in U_\circ$; $f_{xy}(I \times I_y) = I_{xFy}$ and $(f_{yx}(I_y \times I) = I_{yFx})$.

**Theorem 3.5** Let U be a fuzzy left (right) hyperideal of the fuzzy semihypergroup $((H, I), F)$. Then $(U, F)$ is a fuzzy subsemihypergroup *iff* for all $x, y \in U_\circ$,

$$f_{xy}(I \times I_y) = f_{xy}(I_x \times I_y) \quad (f_{yx}(I_y \times I) = f_{yx}(I_y \times I_x)). \tag{25}$$

**Proof 2** Let us consider the case when U is a fuzzy left hyperideal of the fuzzy semihypergroup $((H, I), F)$. This is equivalent to: for all $x, y \in U_\circ$, that

$$f_{xy}(I \times I_y) = f_{xy}(I_x \times I_y),$$

for all $x, y \in U_o$. Then, by Theorem 3.6, we have
(i) $(U_o, F)$ is a left hyperideal of the ordinary semihypergroup $(H, F)$,
(ii) For all $x \in H$, $y \in U_o$ we have $f_{xy}(I \times I_y) = f_{xy}(I_x \times I_y)$.
Then we see that (i) implies $(U_\circ, F)$ is an ordinary subsemihypergroup of $(H, F)$. By (ii) and by Theorem 3.3, we can see that $(U, F)$ is a fuzzy subsemihypergroup of $((H, I), F)$. Conversely, suppose that U is a fuzzy left hyperideal, and it is a fuzzy subsemihypergroup of $((H, I), F)$. Then we have

$$f_{xy}(I \times I_y) = I_{xFy} = f_{xy}(I_x \times I_y), \text{ for all } x, y \in U_\circ. \tag{26}$$

**Corollary 3.6** Let U be a fuzzy hyperideal of the fuzzy semihypergroup $((H, I), F)$. Then $(U, F)$ is a fuzzy subsemihypergroup *iff* the comembership functions $f_{xy}$ satisfy one of the following conditions: for all $x, y \in U_\circ$, we have

$$f_{xy}(I \times I_y) = f_{xy}(I_x \times I_y). \; or \; f_{yx}(I_y \times I) = f_{yx}(I_y \times I_x). \tag{27}$$

In the next theorems, we have some conditions for fuzzy hyperideals induced by a fuzzy subset. Let $A$ be a fuzzy subset of H, and $((H, I), Ð$ is a fuzzy semihypergroup. Then the set $H(A) = \{(x, [0, Ax]) : x \in A_\circ\}$, is the fuzzy subspace induced by $A$, where $A_\circ = \{x \in H : Ax = 0\}$ is the support of $A$. For this special fuzzy subspace $H(A)$, Theorems 3.3, 3.4 and 3.5, can be reformulated in the following form:

**Theorem 3.7** Let H(A) be a fuzzy subspace of the fuzzy semihypergroup $((H, I), F)$. Then $H(A)$ is a fuzzy hyperideal *iff*

(i) $(A_\circ, F)$ is a hyperideal of the ordinary semihypergroup $(H, F)$,

(ii) For all $x \in H$, $y \in A_\circ$, $f_{xy}(1 \times A\, y) = A(xFy)$, $(f_{yx}(A_y \times 1) = A(yFx))$.

## 3.1 The relationship between fuzzy hyperideals and the classical fuzzy hyperideals

We recall that fuzzy semihypergroup $((H, I), F)$, where $F = (F, f_{xy})$, is said to be a uniform fuzzy semihypergroup, if $F = (F, \wedge)$, where $F$ is a fuzzy binary hyperoperation on $(H, I)$ and $\wedge$ is the minimum function from the vector space $I \times I$ into $I$.

**Theorem 3.8** Let $((H, I), F)$ be a uniform fuzzy semihypergroup such that the comembership functions $f_{xy}$ have the t-norm property, and let $A$ be a fuzzy subset of $H$ which induces a fuzzy (left, right) hyperideal $(H(A), F)$ of $((H, I), F)$. Then $A$ is a classical fuzzy hyperideal of the ordinary semihypergroup $(H, F)$.

**Proof 3** Suppose that $A$ is a fuzzy subset of $H$ such that the induced fuzzy subspace $H(A)$ by $A$ is a fuzzy (left, right) hyperideal $(H(A), F)$ of the uniform fuzzy semihypergroup $((H, I), F)$, and let $f_{xy} = \wedge$ have the t-norm property. Then by Theorem 3.9(ii), we have for all $x, y \in A_\circ$,
$f_{xy}(A(x), A(y)) = A(xF\,y)$. Since $A(x) = A(y) = 0$ for each $x, y \notin A_O$, then

$$A(xF\,y) \geq f_{xy}(A(x), A(y)), \tag{28}$$

for all $x, y \in H$, which means that $A$ is a classical fuzzy left hyperideal of the ordinary semihypergroup $(H, F)$.

**Remark 2** The converse of the above Theorem 3.10, is not true, in general, i.e, there is a classical fuzzy hyperideal $A$ of a semihypergroup $(H, F)$ such that a fuzzy subspace $H(A)$ induced by $A$ is not a fuzzy (left, right) hyperideal of the uniform fuzzy semihypergroup $((H, I), F)$ in our sense as we see in the following example.

The next theorem shows the relationship between the introduced fuzzy (left, right) hyperideals in fuzzy semihypergroups based on fuzzy space and the classical fuzzy hyperideal in semihypergroup.

**Theorem 3.9** Let $(Y, F)$ be an ordinary (left,right) hyperideal of the semihypergroup $(H, F)$, and $A$ is a fuzzy subset of $H$ such that $A_O = Y$. Then there is a fuzzy semihypergroup $((H, I), G)$ such that the induced fuzzy subspace $H(A)$ by $A$ is a fuzzy (left, right) hyperideal of $((H, I), G)$.

**Corollary 3.10** Every classical fuzzy (left, right) hyperideal of the semihypergroup $(H, F)$ induces a fuzzy (left, right) hyperideal relative to some fuzzy semihypergroup $((H, I), G)$

**Conclusion**
In this paper, we have generalized the study initiated by Dib (Dib. K. et al, 1949, 1989) about fuzzy ideal based on fuzzy space and Davvaz (Davvaz. B. et al,

 

1998, 2006) and (Davvaz. B, Kazanci. O, 2008) about fuzzy hyperideals in semihypergroups to fuzzy hyperideals in fuzzy semihypergroup based on fuzzy space by Dib, and we introduce a relation between this the fuzzy (left, right) hyperideals and the fuzzy hyperideals in the classical case by Davvaz.

## References

[1] A.Kamali ,B.Davvaz , On the number of fuzzy subgroups of dicyclic Groups, Soft Computing (2020), pp. 6183-6191.

[2] A.Rosenfeld , Fuzzy groups, Journal of Mathematical Analysis and Application (1971), pp. 512-517.

[3] B.Davvaz, On $H_V$ -rings and fuzzy $H_V$-ideal, J. Fuzzy Math (1998), pp. 33-42.

[4] B.Davvaz ,1999, Fuzzy $H_V$ –groups, Fuzzy Sets and Systems (1999), pp. 191-195.

[5] .B.Davvaz, M. Fathi , A.Salleh , Fuzzy hypergroup based on fuzzy universal sets, Information Sciences(2010), pp. 3021-3032.

[6] [B.Davvaz, M. Fathi ,A. Salleh, Fuzzy hyperrings based on fuzzy universal sets, Information Sciences (2010), pp. 3033-3043.

[7] F.Marty Sur une generalization de la notion de group, Proceedins of the 8th Congress Mathematics Scandenaves, Stockholm (1934), pp. 45-49.

[8] .J.Liu, Fuzzy invariant subgroups and fuzzy ideals, Fuzzy sets and systems(1982), pp. 133-139.

[9] K.Abdulmula, The relationship between intuitionistic fuzzy normal subhypergroups based on intuitionistic fuzzy space and Atanassove's approach, The Center of Research and Scientific Consultancy, University Bulletin (2021), pp. 91-118.

[10] K. Abdulmula, B. Davvaz , A. Salleh , Atanassove's intuitionistic fuzzy hyperrings (rings) based on intuitionistic fuzzy universal sets, information Sciences.(2013), pp.407-438.

[11] K.Asghar ,B. Davvaz, Fuzzy hyperideals of hyperquantale, Journal of Intelligent and Fuzzy Systems.(2019 ), pp.5605-5615.

[12] .K.Dib , On fuzzy spaces and fuzzy group theory, Information Sciences (1994), pp. 253-282.

[13] K.Dib, N.Galhum, The fuzzy ideals and fuzzy bi-ideals in fuzzy semigroups, Fuzzy sets and systems (1996 ), pp.103-111.

[14] K.Rajesh, Fuzzy ideals and fuzzy semiprime ideals: some ring theoretic analogues, Information Sciences (1992), pp. 43-52.

[15] L.Guilong, Fuzzy invariant subgroups and fuzzy ideals. Fuzzy sets and systems (2006), pp. 133-139.

[16] L.Guilong , Generalized rough sets over fuzzy lattices, Information Sciences (2007), pp. 1651-1662.

[17] L.Zadeh, Fuzzy sets, Inform. Control (1965), pp. 338-353.

[18] M.Fathi, Intuitionistic fuzzy algebra and fuzzy hyperalgebra, PhD thesis. Universiti Kebangsaan Malaysia (2010).

[19] M.Sen, A.Reza, G. Chowdhury, Fuzzy hypersemigroups, Soft Computing A Fusion of Foundations. Methodologies and Applications (2008), pp. 891-900.

[20] M.Xueling, Z.Jianming, B.Davvaz, B.Young, Some kinds of (C,CCq) Intervalvalued fuzzy ideals of BCI-algebras,Information Sciences (2008), pp. 3738-3754.

[21] N. Hassan, Dib. K, The fuzzy ring and fuzzy Ideals, Fuzzy Math (1998) pp.245-261.

[22] [22] .O.Kazanci , B. Davvaz , On the structure of rough prime (primary) ideals and rough fuzzy prime (primary) ideals in commutative rings, Information Sciences (2008), pp. 1343-1354.

[23] O.Kazanci, S. Yamak, Fuzzy ideals and semiprime fuzzy ideals in Semigroups, Information Sciences (2009), pp. 3720-3731.

[24] P. Corsini, Fuzzy sets and join spaces associated with rough sets, . Circolo Matematico di Palermo (2002),pp. 253-282.

[25] P.Davvaz, Roughness based on fuzzy ideals, Information Sciences. (2006), pp.2417-2437.

[26] T.Marius, Distributivity in lattices of fuzzy subgroups, Information Sciences (2009), pp. 1163-1168.

[27] V.Leoreanu-Fotea, A new type of fuzzy n-ary hyperstructures, Information Sciences (2009), pp. 2710-2718.

[28] V.Leoreanu-Fotea, B. Davvaz , Roughness in n-ary hypergroups, Information Sciences (2008), pp. 4114-4124.